Analysis of the vector and axialvector B_c mesons with QCD sum rules

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Abstract

In this article, we study the vector and axialvector B_c mesons using the QCD sum rules, and make reasonable predictions for the masses and decay constants, then calculate the leptonic decay widths, the present predictions can be confronted with the experimental data in the future.

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1 Introduction

In 1998, the CDF collaboration observed the pseudoscalar bottom-charm B_c mesons through the decay modes $B_c^\pm \to J/\psi\ell^\pm X$ and $B_c^\pm \to J/\psi\ell^\pm\bar{\nu}_\ell$ in $p\bar{p}$ collisions at the energy $\sqrt{s}=1.8\,\mathrm{TeV}$ at the Fermilab Tevatron, the measured mass is $M_{B_c}=(6.40\pm0.39\pm0.13)\,\mathrm{GeV}$ [1]. In 2007, the CDF collaboration observed the pseudoscalar B_c mesons with a significance exceeds $8\,\sigma$ through the decay modes $B_c^\pm \to J/\psi\pi^\pm$ in $p\bar{p}$ collisions at the energy $\sqrt{s}=1.96\,\mathrm{TeV}$ using the Collider Detector at Fermilab (CDF II), the measured mass is $M_{B_c}=(6275.6\pm2.9\pm2.5)\,\mathrm{MeV}$ [2]. In 2008, the D0 collaboration reconstructed the $B_c^\pm \to J/\psi\pi^\pm$ decays and observed the pseudoscalar B_c mesons with a significance more than $5\,\sigma$, the measured mass is $M_{B_c}=(6300\pm14\pm5)\,\mathrm{MeV}$ [3]. Now the average value listed in the Review of Particle Physics is $M_{B_c}=(6.277\pm0.006)\,\mathrm{GeV}$ [4]. Other B_c mesons, such as the scalar, vector, axialvector, tensor B_c mesons, have not been observed yet, but they are expected to be produced at the Large Hadron Collider (LHC) in the future [5].

The heavy quarkonium states and triply-heavy baryon states play an important role both in studying the interplays between the perturbative and nonperturbative QCD and in understanding the heavy quark dynamics due to the absence of the light quark contaminations. The bottom-charm quarkonium states B_c, which consist of the heavy quarks with different flavors, are of special interesting. The ground states and the excited states lying below the BD, BD^*, B^*D, B^*D^* thresholds cannot annihilate into gluons, and therefore are more stable than the corresponding charmonium and bottomonium states, and would have widths less than a hundred KeV [6]. The excited states can undergo radiative or hadronic transitions to the ground state pseudoscalar B_c mesons, which decay weakly. There have been several theoretical works on the mass spectroscopy of the B_c mesons, such as the relativized (or relativistic) quark model with an special phenomenological potential [6, 7, 8, 9], the nonrelativistic quark model with an special phenomenological potential [10, 11, 12], the semi-relativistic quark model using the shifted large-N expansion [13], the perturbative QCD [14], the nonrelativistic renormalization group [15], the lattice QCD [16, 17], etc.

The QCD sum rules is a powerful theoretical tool in studying the heavy quarkonium states [18, 19], and the existing works focus on the S-wave heavy quarkonium states J/ψ, η_c, Υ, η_b, and the P-wave spin-triplet heavy quarkonium states χ_{cj}, χ_{bj}, $j=0,1,2$ [19, 20]. The pseudoscalar B_c mesons have been studied by the full QCD sum rules [21, 22, 23, 24] and the potential approach combined with the QCD sum rules [11, 25, 26], while the vector B_c mesons (B_c^*) have been studied by the full QCD sum rules [23, 24], and the axialvector B_c mesons have not been studied yet. In Ref.[23], Colangelo, Nardulli and Paver took the leading-order $\mathcal{O}(1)$ approximation, obtained the values $M_{B_c}\approx 6.35\,\mathrm{GeV}$ and $f_{B_c^*}\approx f_{B_c}=(360\pm60)\,\mathrm{MeV}$, and did not present the value $M_{B_c^*}$. In Ref.[24], Narison took into account the next-to-leading-order $\mathcal{O}(\alpha_s)$ contributions by

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assuming that one quark had zero mass, obtained the values $M_{B_c^*} - M_{B^*} = (1.53 \pm 0.18)\,\mathrm{GeV}$, $f_{B_c^*} = \frac{\sqrt{2}M_{B_c^*}}{2\gamma_{B_c^*}}$ and $\gamma_{B_c^*} = 14.0 \pm 1.0$, the mass $M_{B_c^*} = M_{B^*} + (1.53 \pm 0.18)\,\mathrm{GeV}$ is much larger than other theoretical calculations [6, 7, 8, 9, 10, 11, 12, 16]. Those studies based on the QCD sum rules were preformed before the pseudoscalar B_c mesons were observed by the CDF collaboration, the predictions should be updated. Now we can take the experimental data as guides to choose the suitable Borel parameters and continuum threshold parameters. Naively, we expect that the masses of the pseudoscalar, vector and axialvector B_c mesons have the hierarchy: $M_{B_c(0^-)} < M_{B_c(1^-)} < M_{B_c(1^+)}$, the 0^-, 1^- and 1^+ denote the spin-parity J^P. Furthermore, the calculations based on the nonrelativistic renormalization group indicate that $M_{B_c(1^-)} - M_{B_c(0^-)} = (50 \pm 17^{+15}_{-12})\,\mathrm{MeV}$ [15]. In this article, we recalculate the masses and decay constants of the vector and axialvector B_c mesons with the QCD sum rules by including the next-to-leading-order $\mathcal{O}(\alpha_s)$ contributions, and make predictions for the leptonic decay widths. The decay constants are basic parameters in studying the exclusive processes of the B_c mesons.

The article is arranged as follows: we derive the QCD sum rules for the masses and decay constants of the vector and axialvector B_c mesons in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusions.

2 QCD sum rules for the vector and axialvector B_c mesons

In the following, we write down the two-point correlation functions $\Pi_{\mu\nu}(p)$ in the QCD sum rules,

$$\Pi_{\mu\nu}(p) = i\int d^4x e^{ip\cdot x}\langle 0|T\left\{J_\mu(x)J_\nu^\dagger(0)\right\}|0\rangle\,, \tag{1}$$

$$\begin{aligned} J_\mu^V(x) &= \bar{c}(x)\gamma_\mu b(x)\,, \\ J_\mu^A(x) &= \bar{c}(x)\gamma_\mu\gamma_5 b(x)\,, \end{aligned} \tag{2}$$

where $J_\mu(x) = J_\mu^V(x), J_\mu^A(x)$, the vector and axialvector currents $J_\mu^V(x)$ and $J_\mu^A(x)$ interpolate the vector and axialvector B_c mesons, respectively.

We can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J_\mu(x)$ into the correlation functions $\Pi_{\mu\nu}(p)$ to obtain the hadronic representation [18, 19]. After isolating the ground state contributions from the vector and axialvector B_c mesons, we get the following result,

$$\begin{aligned} \Pi_{\mu\nu}(p) &= \frac{f_{B_c}^2 M_{B_c}^2}{M_{B_c}^2 - p^2}\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right) + \cdots \\ &= \Pi(p)\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right) + \cdots\,, \end{aligned} \tag{3}$$

where the decay constants f_{B_c} are defined by

$$\langle 0|J_\mu(0)|B_c(p)\rangle = f_{B_c} M_{B_c} \varepsilon_\mu\,, \tag{4}$$

and the ε_μ are the polarization vectors of the vector and axialvector B_c mesons. The $\Pi(p)$ can be expressed in the following form through the dispersion relation,

$$\Pi(p) = \int_{(m_b+m_c)^2}^{\infty} ds \frac{f_{B_c}^2 M_{B_c}^2 \delta(s - M_{B_c}^2)}{s - p^2} + \cdots\,. \tag{5}$$

Now, we briefly outline the operator product expansion for the correlation functions $\Pi_{\mu\nu}(p)$ in perturbative QCD. We contract the quark fields in the correlation functions $\Pi_{\mu\nu}^{V/A}(p)$ (here we

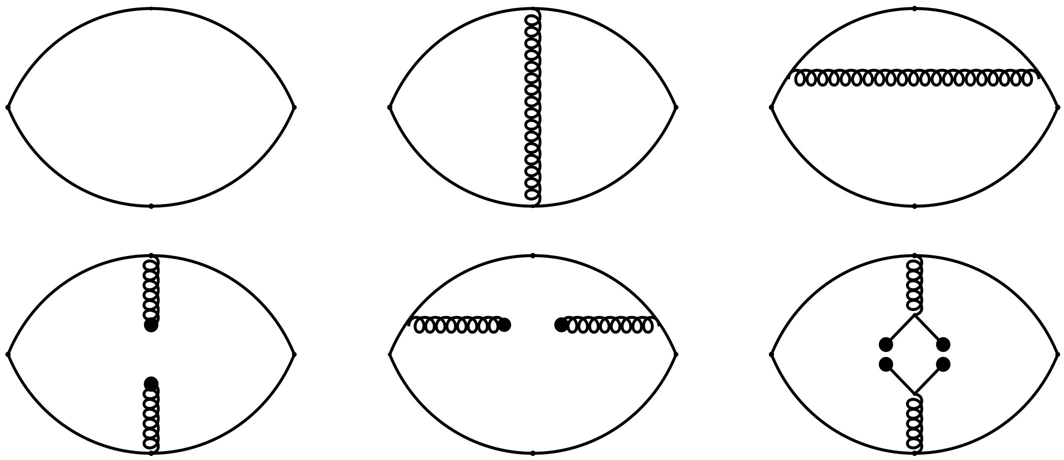

Figure 1: The typical diagrams we calculate in the operator product expansion.

add the indexes V and A to denote the vector and axialvector currents respectively) with Wick theorem firstly,

$$\begin{aligned}
\Pi^V_{\mu\nu}(p) &= -i\int d^4x e^{ip\cdot x} Tr\left\{\gamma_\mu B_{ij}(x)\gamma_\nu C_{ji}(-x)\right\} \, , \\
\Pi^A_{\mu\nu}(p) &= -i\int d^4x e^{ip\cdot x} Tr\left\{\gamma_\mu \gamma_5 B_{ij}(x)\gamma_\nu \gamma_5 C_{ji}(-x)\right\} \, ,
\end{aligned}$$

where the $B_{ij}(x)$ and $C_{ij}(x)$ are the full heavy quark propagators, and can be written as $S_{ij}(x)$ collectively,

$$\begin{aligned}
S_{ij}(x) &= \frac{i}{(2\pi)^4}\int d^4k e^{-ik\cdot x}\left\{\frac{\delta_{ij}}{\not{k}-m_Q} - \frac{g_s G^n_{\alpha\beta}t^n_{ij}}{4}\frac{\sigma^{\alpha\beta}(\not{k}+m_Q)+(\not{k}+m_Q)\sigma^{\alpha\beta}}{(k^2-m_Q^2)^2} + \frac{\delta_{ij}\langle g_s^2 GG\rangle}{12}\right. \\
&\left. \frac{m_Q k^2+m_Q^2\not{k}}{(k^2-m_Q^2)^4} + \frac{g_s D_\alpha G^n_{\beta\lambda}t^n_{ij}}{3}\frac{(\not{k}+m_Q)(f^{\lambda\beta\alpha}+f^{\lambda\alpha\beta})(\not{k}+m_Q)}{(k^2-m_Q^2)^4} + \cdots\right\} \, , \\
f^{\lambda\alpha\beta} &= \gamma^\lambda(\not{k}+m_Q)\gamma^\alpha(\not{k}+m_Q)\gamma^\beta \, , \qquad (6)
\end{aligned}$$

and $t^n = \frac{\lambda^n}{2}$, the λ^n is the Gell-Mann matrix, the i, j are color indexes, $D_\alpha = \partial_\alpha - ig_s G^n_\alpha t^n$, and the $\langle g_s^2 GG\rangle = \langle g_s^2 G^{\alpha\beta}_n G^n_{\alpha\beta}\rangle$ is the gluon condensate [19]; then complete the integrals both in the coordinate space and in the momentum space; finally obtain the correlation functions $\Pi_{\mu\nu}(p)$ (or $\Pi(p)$) at the level of the quark-gluon degrees of freedom. Furthermore, we calculate the $\mathcal{O}(\alpha_s)$ corrections to the perturbative terms originally. The typical Feynman diagrams we calculate in the operator product expansion are shown in Fig.1.

Once analytical expressions of the correlation functions at the quark level are obtained and expressed as $\int_{(m_b+m_c)^2}^\infty ds \frac{1}{s-p^2}\left[\rho_\pm(s)+\frac{\alpha_s(\mu)}{\pi}\rho^{\alpha_s}_\pm(s)\right]$ through the dispersion relation, then we can take the quark-hadron duality and perform the Borel transforms with respect to the variable $P^2=-p^2$ to obtain the QCD sum rules,

$$f^2_{B_c(1^\mp)}M^2_{B_c(1^\mp)}\exp\left(-\frac{M^2_{B_c(1^\mp)}}{T^2}\right) = \int_{(m_b+m_c)^2}^{s_0} ds\left[\rho_\pm(s)+\frac{\alpha_s(\mu)}{\pi}\rho^{\alpha_s}_\pm(s)\right]\exp\left(-\frac{s}{T^2}\right) \, , \quad (7)$$

where

$$
\begin{aligned}
\rho_{\pm}(s) \;\;=\;\; & \frac{3}{4\pi^2}\int_{x_i}^{x_f} dx\left[x(1-x)(2s-\widetilde{m}_Q^2)\pm m_b m_c\right] \mp \frac{m_b m_c}{24T^4}\langle\frac{\alpha_s GG}{\pi}\rangle\int_0^1 dx\left[\frac{m_c^2}{x^3}+\frac{m_b^2}{(1-x)^3}\right] \\
& \delta(s-\widetilde{m}_Q^2) \pm \frac{m_b m_c}{8T^2}\langle\frac{\alpha_s GG}{\pi}\rangle\int_0^1 dx\left[\frac{1}{x^2}+\frac{1}{(1-x)^2}\right]\delta(s-\widetilde{m}_Q^2) \\
& -\frac{s}{24T^4}\langle\frac{\alpha_s GG}{\pi}\rangle\int_0^1 dx\left[\frac{(1-x)m_c^2}{x^2}+\frac{x m_b^2}{(1-x)^2}\right]\delta(s-\widetilde{m}_Q^2) \\
& -\frac{1}{12}\langle\frac{\alpha_s GG}{\pi}\rangle\int_0^1 dx\left[1+\frac{s}{2T^2}\right]\delta(s-\widetilde{m}_Q^2) + \frac{4\alpha_s^2\langle\bar{q}q\rangle^2}{81T^2}\int_0^1 dx\left[\frac{2}{x(1-x)}+\frac{m_b^2 m_c^2}{x^2(1-x)^2T^4}\right. \\
& \left.+\frac{m_b^2+m_c^2\pm 9m_b m_c}{3x(1-x)T^2}-\frac{2}{3}\left(1+\frac{s}{T^2}-\frac{s^2}{T^4}\right)+\frac{5s}{3x(1-x)T^2}\right]\delta(s-\widetilde{m}_Q^2)\,, \qquad (8)
\end{aligned}
$$

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\begin{aligned}
\rho^{\alpha_s}_{\pm}(s) \;\;=\;\; & \frac{1}{2\pi^2}\int_{m_b^2}^{(\sqrt{s}-m_c)^2} dt\int_{z_i}^{z_f} dz\frac{1}{sz\lambda^{\frac{1}{2}}(s,t,m_c^2)}\left\{\lambda^{\frac{1}{2}}(s,t,m_c^2)\left[z(m_c^2\mp 2m_b m_c+t-2s)\right.\right. \\
& \left.-(z+1)(t\mp m_b m_c-s)\right]-\lambda^{\frac{1}{2}}(s,m_b^2,m_c^2)\left[z(m_c^2\mp 2m_b m_c+m_b^2-2s)\right. \\
& \left.\left.-(z+1)(m_b^2\mp m_b m_c-s)\right]\right\} \\
& +\frac{1}{2\pi^2}\int_{m_b^2}^{(\sqrt{s}-m_c)^2} dt\int_{z_i}^{z_f} dz\frac{(t-m_b^2)\left[tz^2+m_c^2+z(t+m_c^2-s)\right]}{sz\lambda^{\frac{3}{2}}(s,t,m_c^2)} \\
& \left\{\lambda^{\frac{1}{2}}(s,t,m_c^2)(s-t\pm 2m_b m_c-m_c^2)-\lambda^{\frac{1}{2}}(s,m_b^2,m_c^2)(s-m_b^2\pm 2m_b m_c-m_c^2)\right\} \\
& +\frac{1}{12\pi^2}\int_{m_b^2}^{(\sqrt{s}-m_c)^2} dt\int_{z_i}^{z_f} dz\frac{(t-m_b^2)}{s^2z\lambda^{\frac{3}{2}}(s,t,m_c^2)}\left[1-4z+z^2-\frac{6(tz^2+m_c^2)}{t+m_c^2-s}\right. \\
& \left.+6(t+m_c^2)\frac{z(t+m_c^2-s)+tz^2+m_c^2}{\lambda(s,t,m_c^2)}-24tm_c^2\frac{z(t+m_c^2-s)+tz^2+m_c^2}{\lambda(s,t,m_c^2)(t+m_c^2-s)}\right] \\
& \left\{\lambda^{\frac{3}{2}}(s,t,m_c^2)(m_c^2\mp 2m_b m_c+t-s)-\lambda^{\frac{3}{2}}(s,m_b^2,m_c^2)(m_c^2\mp 2m_b m_c+m_b^2-s)\right\} \\
& +\frac{1}{12\pi^2}\int_{m_b^2}^{(\sqrt{s}-m_c)^2} dt\int_{z_i}^{z_f} dz\frac{\left[z(m_c^2-s-t)-(t-m_c^2-s)\right]}{s^2z\lambda^{\frac{3}{2}}(s,t,m_c^2)} \\
& \left\{\left[2(m_b^2+m_c^2-s)+(1-z)(t-m_b^2)\right]\left[\lambda^{\frac{3}{2}}(s,t,m_c^2)-\lambda^{\frac{3}{2}}(s,m_b^2,m_c^2)\right]\right\} \\
& +\frac{1}{2\pi^2}\int_{m_b^2}^{(\sqrt{s}-m_c)^2} dt\int_{z_i}^{z_f} dz\frac{1}{sz(t-m_b^2)\lambda^{\frac{1}{2}}(s,t,m_c^2)} \\
& \left\{\lambda^{\frac{1}{2}}(s,t,m_c^2)\left[t^2+m_c^4\mp 2m_b m_c^3+2m_b^2m_c^2\mp 2m_b m_c t-2s(t+m_c^2\mp m_b m_c)+s^2\right.\right. \\
& \left.+\frac{\lambda(s,t,m_c^2)(m_b^2+m_c^2-s)}{3s}\right]-\lambda^{\frac{1}{2}}(s,m_b^2,m_c^2)\left[m_b^4+m_c^4\mp 2m_b m_c^3+2m_b^2m_c^2\right. \\
& \left.\left.\mp 2m_b^3m_c-2s(m_b^2+m_c^2\mp m_b m_c)+s^2+\frac{\lambda(s,m_b^2,m_c^2)(m_b^2+m_c^2-s)}{3s}\right]\right\}
\end{aligned}
$$

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\begin{aligned}
&+\frac{1}{4\pi^2}\int_{m_c^2}^{(\sqrt{s}-m_b)^2}dt\frac{(m_b\mp m_c)^2-s}{ts}\left\{\lambda^{\frac{1}{2}}(s,t,m_b^2)-\lambda^{\frac{1}{2}}(s,m_b^2,m_c^2)\right\}\\
&+\frac{1}{8\pi^2}\int_{m_c^2}^{(\sqrt{s}-m_b)^2}dt\frac{t-m_c^2}{t^2s}\left\{\lambda^{\frac{1}{2}}(s,t,m_b^2)\left[t+s-m_b^2-\frac{\lambda(s,t,m_b^2)}{3s}\right]\right.\\
&\left.-\lambda^{\frac{1}{2}}(s,m_b^2,m_c^2)\left[m_c^2+s-m_b^2-\frac{\lambda(s,m_b^2,m_c^2)}{3s}\right]\right\}\\
&+\frac{1}{4\pi^2}\int_{m_c^2}^{(\sqrt{s}-m_b)^2}dt\frac{1}{ts(t-m_c^2)}\left\{\lambda^{\frac{1}{2}}(s,t,m_b^2)\left[t(s-m_b^2-t)\right.\right.\\
&\left.+3m_c^2(t+m_b^2-s)\mp 4m_c^3m_b+\frac{2m_c^2\lambda(s,t,m_b^2)}{3s}\right]-\lambda^{\frac{1}{2}}(s,m_b^2,m_c^2)\\
&\left.\left[m_c^2(s-m_b^2-m_c^2)+3m_c^2(m_c^2+m_b^2-s)\mp 4m_c^3m_b+\frac{2m_c^2\lambda(s,m_b^2,m_c^2)}{3s}\right]\right\}\\
&+\frac{1}{4\pi^2}\int_{m_b^2}^{(\sqrt{s}-m_c)^2}dt\frac{(m_b\mp m_c)^2-s}{ts}\left\{\lambda^{\frac{1}{2}}(s,t,m_c^2)-\lambda^{\frac{1}{2}}(s,m_b^2,m_c^2)\right\}\\
&+\frac{1}{8\pi^2}\int_{m_b^2}^{(\sqrt{s}-m_c)^2}dt\frac{t-m_b^2}{t^2s}\left\{\lambda^{\frac{1}{2}}(s,t,m_c^2)\left[t+s-m_c^2-\frac{\lambda(s,t,m_c^2)}{3s}\right]\right.\\
&\left.-\lambda^{\frac{1}{2}}(s,m_b^2,m_c^2)\left[m_b^2+s-m_c^2-\frac{\lambda(s,m_b^2,m_c^2)}{3s}\right]\right\}\\
&+\frac{1}{4\pi^2}\int_{m_b^2}^{(\sqrt{s}-m_c)^2}dt\frac{1}{ts(t-m_b^2)}\left\{\lambda^{\frac{1}{2}}(s,t,m_c^2)\left[t(s-m_c^2-t)\right.\right.\\
&\left.+3m_b^2(t+m_c^2-s)\mp 4m_b^3m_c+\frac{2m_b^2\lambda(s,t,m_c^2)}{3s}\right]-\lambda^{\frac{1}{2}}(s,m_b^2,m_c^2)\\
&\left.\left[m_b^2(s-m_c^2-m_b^2)+3m_b^2(m_b^2+m_c^2-s)\mp 4m_b^3m_c+\frac{2m_b^2\lambda(s,m_b^2,m_c^2)}{3s}\right]\right\}\,, \qquad (9)
\end{aligned}
$$

$\widetilde{m}_Q^2=\frac{m_b^2}{1-x}+\frac{m_c^2}{x}$, $x_{f/i}=\frac{1}{2s}\left[s+m_c^2-m_b^2\pm\sqrt{\lambda(s,m_b^2,m_c^2)}\right]$, $z_{f/i}=\frac{1}{2t}\left[s-t-m_c^2\pm\sqrt{\lambda(s,t,m_c^2)}\right]$, $\lambda(a,b,c)=a^2+b^2+c^2-2ab-2ac-2bc$, the $\rho_{+}^{(\alpha_s)}(s)$ and $\rho_{-}^{(\alpha_s)}(s)$ are the QCD spectral densities of the vector and axialvector B_c mesons respectively, the T^2 is the Borel parameter, and the s_0 are the continuum threshold parameters. In this article, we take into account the contributions from the gluon condensate $\langle\frac{\alpha_sGG}{\pi}\rangle$ and the four quark condensates $\langle\bar{q}\gamma_\mu t^aq\bar{q}\gamma^\mu t^aq\rangle$, and neglect the contributions from the three-gluon condensate due to its small value. In calculations, we have used the equation of motion, $D^\nu G^a_{\mu\nu}=\sum_{q=u,d,s}g_s\bar{q}\gamma_\mu t^aq$, and assumed vacuum saturation for the four quark condensates and taken the approximation $\langle\bar{s}s\rangle=\langle\bar{q}q\rangle$.

We calculate the imaginary parts $\frac{\alpha_s(\mu)}{\pi}\rho^{\alpha_s}_{\pm}(s)$ of the $\mathcal{O}(\alpha_s)$ corrections with the Cutkosky's rule firstly [27], then use the dispersion relation to obtain the full $\mathcal{O}(\alpha_s)$ corrections. The calculations should be carried out in the D-dimension, because there exist divergences in the integrals $\frac{\alpha_s(\mu)}{\pi}\int_{(m_b+m_c)^2}^{\infty}ds\frac{1}{s-p^2}\rho^{\alpha_s}_{\pm}(s)$. In calculations, we observe that the divergences (endpoint divergences) are canceled out with each other in the imaginary parts $\frac{\alpha_s(\mu)}{\pi}\rho^{\alpha_s}_{\pm}(s)$, the imaginary parts $\frac{\alpha_s(\mu)}{\pi}\rho^{\alpha_s}_{\pm}(s)$ have no divergences themselves, divergences only occur in the integrals over s. The integrals $\frac{\alpha_s(\mu)}{\pi}\int_{(m_b+m_c)^2}^{s_0}ds\rho^{\alpha_s}_{\pm}(s)\exp\left(-\frac{s}{T^2}\right)$ are convergent and we can take the dimension $D=4$ safely. The terms like $\log\frac{m_b^2}{\mu^2}$ and $\log\frac{m_c^2}{\mu^2}$ do not appear. In this article, we carry out the calculations by assuming $m_c\neq 0$, the perturbative $\mathcal{O}(\alpha_s)$ corrections to the corresponding correlation functions

$\Pi(p)$ of the heavy-light currents cannot be reduced by simply setting $m_c = 0$ [28], as there appear additional endpoint divergences, we should take care of the subtleties in performing the dispersion relation and recalculate the Feynman diagrams [27]. There exist three typical energy-scales m_b^2, m_c^2 and p^2, it is difficult to use the usual procedure of the dimensional regularization (through Feynman parameters) to carry out the two-loop divergent integrals.

We can eliminate the decay constants $f_{B_c(1^\mp)}$ and obtain the QCD sum rules for the masses of the vector and axialvector B_c mesons,

$$M^2_{B_c(1^\mp)} = \frac{\int_{(m_b+m_c)^2}^{s_0} ds \frac{d}{d(-1/T^2)} \left[\rho_\pm(s) + \frac{\alpha_s(\mu)}{\pi}\rho_\pm^{\alpha_s}(s)\right] \exp\left(-\frac{s}{T^2}\right)}{\int_{(m_b+m_c)^2}^{s_0} ds \left[\rho_\pm(s) + \frac{\alpha_s(\mu)}{\pi}\rho_\pm^{\alpha_s}(s)\right] \exp\left(-\frac{s}{T^2}\right)}, \tag{10}$$

then use the resulting masses as input parameters to obtain the decay constants $f_{B_c(1^\mp)}$ from Eq.(7).

3 Numerical results and discussions

The mass of the pseudoscalar B_c meson is $M_{B_c} = (6.277 \pm 0.006)\,\text{GeV}$ from the Particle Data Group [4], while the calculations based on the nonrelativistic renormalization group indicate that $M_{B_c(1^-)} - M_{B_c(0^-)} = (50 \pm 17^{+15}_{-12})\,\text{MeV}$ [15]. We can tentatively take the continuum threshold parameters as $s^0_{B_c(1^-)} = (41-47)\,\text{GeV}^2$ and $s^0_{B_c(1^+)} = (46-54)\,\text{GeV}^2$, and search for the ideal values, where we have assumed that an additional P-wave results in mass-shift about $0.5\,\text{GeV}$ and the energy gap between the ground states and the first radial excited states is about $0.5\,\text{GeV}$.

The vacuum condensates are taken to be the standard values $\langle\bar{q}q\rangle = -(0.24 \pm 0.01\,\text{GeV})^3$ at the energy scale $\mu = 1\,\text{GeV}$ [29]. The quark condensate evolves with the renormalization group equation, $\langle\bar{q}q\rangle(\mu^2) = \langle\bar{q}q\rangle(Q^2)\left[\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right]^{\frac{4}{9}}$. The value of the gluon condensate $\langle\frac{\alpha_s GG}{\pi}\rangle$ has been updated from time to time, and changes greatly [20], we use the recently updated value $\langle\frac{\alpha_s GG}{\pi}\rangle = (0.022 \pm 0.004)\,\text{GeV}^4$ [30].

In this article, we take the $\overline{MS}$ masses $m_c(m_c^2) = (1.275 \pm 0.025)\,\text{GeV}$ and $m_b(m_b^2) = (4.18 \pm 0.03)\,\text{GeV}$ from the Particle Data Group [4]. Furthermore, we take into account the energy-scale dependence of the $\overline{MS}$ masses from the renormalization group equation,

$$\begin{aligned}
m_c(\mu^2) &= m_c(m_c^2)\left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right]^{\frac{12}{25}}, \\
m_b(\mu^2) &= m_b(m_b^2)\left[\frac{\alpha_s(\mu)}{\alpha_s(m_b)}\right]^{\frac{12}{23}}, \\
\alpha_s(\mu) &= \frac{1}{b_0 t}\left[1 - \frac{b_1}{b_0^2}\frac{\log t}{t} + \frac{b_1^2(\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2}\right],
\end{aligned} \tag{11}$$

where $t = \log\frac{\mu^2}{\Lambda^2}$, $b_0 = \frac{33-2n_f}{12\pi}$, $b_1 = \frac{153-19n_f}{24\pi^2}$, $b_2 = \frac{2857-\frac{5033}{9}n_f+\frac{325}{27}n_f^2}{128\pi^3}$, $\Lambda = 213\,\text{MeV}$, $296\,\text{MeV}$ and $339\,\text{MeV}$ for the flavors $n_f = 5$, 4 and 3, respectively [4]. The typical energy scale in the $b\bar{c}$ or $c\bar{b}$ system is $\mu = \sqrt{m^2_{B_c(0^-)} - (m_b + m_c)^2} \approx (2.6 \pm 0.1)\,\text{GeV}$, where the spin-parity 0^- denotes the observed pseudoscalar meson B_c [4], we can take $n_f = 4$. In Fig.2, we plot the threshold $(m_b + m_c)^2$ with variations of the energy scales. From the figure, we can see that the threshold $(m_b + m_c)^2$ decreases quickly with increase of the energy scale, the energy scale should be larger than $1.7\,\text{GeV}$ for the $b\bar{c}$ or $c\bar{b}$ system. The typical energy scale $\mu = (2.5-2.7)\,\text{GeV}$ leads to a rather large integral range $s_0 - (m_b + m_c)^2$, it is a reasonable choice.

In calculations, we observe that the ideal parameters are $T^2 = (4.2-6.2)\,\text{GeV}^2$ [$(4.6-6.6)\,\text{GeV}^2$] and $s_0 = (47 \pm 1)\,\text{GeV}^2$ [$(54 \pm 1)\,\text{GeV}^2$] for the vector [axialvector] B_c mesons, the corresponding

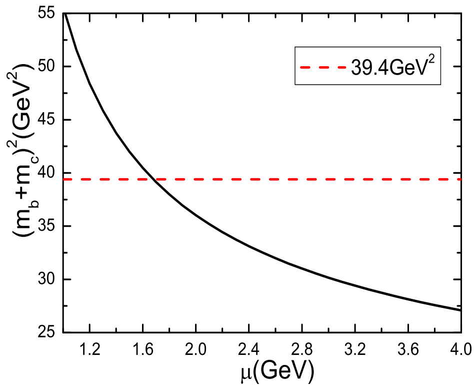

Figure 2: The energy scale dependence of the threshold $(m_b + m_c)^2$, where $39.4\,\text{GeV}^2$ is the squared mass of the pseudoscalar meson B_c.

	$T^2(\text{GeV}^2)$	$s_0(\text{GeV}^2)$	pole	$M(\text{GeV})$	$f_{B_c}(\text{GeV})$
$B_c(1^-)$	$4.2-6.2$	47 ± 1	$(51-88)\%$	6.34 ± 0.09	0.79 ± 0.10
$B_c(1^+)$	$4.6-6.6$	54 ± 1	$(51-85)\%$	6.74 ± 0.10	0.81 ± 0.11

Table 1: The Borel parameters, continuum threshold parameters, pole contributions, masses and decay constants of the vector and axialvector B_c mesons.

pole contributions and the resulting masses and decay constants are presented in Table 1 and Fig.3. From the table, we can see that the energy scale and the Borel parameters are of the same order, $\mu^2 = \mathcal{O}(T^2)$. The threshold parameters and the predicted masses satisfy the relations $\sqrt{s^0_{B_c(1^-)}} - M_{B_c(1^-)} \approx 0.5\,\text{GeV}$ and $\sqrt{s^0_{B_c(1^+)}} - M_{B_c(1^+)} \approx 0.6\,\text{GeV}$, which are compatible with our naive expectation that the energy gap between the ground state and first radial excited is about $(0.5-0.7)\,\text{GeV}$. From the table, we can also see that the uncertainties of the masses are much smaller than that of the decay constants, because the masses are obtained from a ratio, see Eq.(10), some uncertainties in the numerator and denominator are canceled out with each other.

The calculations based on the nonrelativistic renormalization group indicate that $M_{B_c(1^-)} - M_{B_c(0^-)} = (50 \pm 17^{+15}_{-12})\,\text{MeV}$ [15], the present prediction $M_{B_c(1^-)} - M_{B_c(0^-)} = 63\,\text{MeV}$ based on the $\overline{MS}$ masses plus $\mathcal{O}(\alpha_s)$ corrections is satisfactory, here we take the central value.

In Table 2, we present the theoretical values of the vector and axialvector B_c mesons from the relativized (or relativistic) quark model with an special potential [6, 7, 8, 9], the nonrelativistic quark model with an special potential [10, 11, 12], and the lattice QCD [16]. From the Table, we can see that the present predictions are consistent with those values. In Table 3, we present the values of the decay constants of the vector and axialvector B_c mesons from the relativistic quark model with an special potential [7], the nonrelativistic quark model with an special potential [10, 11, 12], the light-front quark model [31, 32], the Bethe-Salpeter equation [33], and field correlator method [34]. Compared with those theoretical calculations $f_{B_c(1^-)} = (380-520)\,\text{MeV}$ [7, 10, 11, 12, 31, 32, 33, 34], the present prediction $f_{B_c(1^-)} = (790\pm100)\,\text{MeV}$ is somewhat larger, while the present prediction $f_{B_c(1^+)} = (810\pm110)\,\text{MeV}$ is much larger than the value 160 MeV from the Bethe-Salpeter equation [33]. At present time, it is difficult to say which value is superior to others.

	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[16]	This work
$B_c(1^-)$	6.338	6.332	6.308	6.340	6.341	6.317	6.337	6.321	6.34 ± 0.09
$B_c(1^+)$	6.741	6.734	6.738	6.730	6.737	6.717	6.730	6.743	6.74 ± 0.10

Table 2: The masses of the vector and axialvector B_c mesons from different theoretical approaches, the unit is GeV.

	[7]	[10]	[11]	[12]	[31]	[32]	[33]	[34]	This work
$B_c(1^-)$	503	517	460	500	398	387	418	453	790 ± 100
$B_c(1^+)$							160		810 ± 110

Table 3: The decay constants of the vector and axialvector B_c mesons from different theoretical approaches, the unit is MeV.

The leptonic decay widths $\Gamma_{\ell\bar{\nu}_\ell}$ of the vector and axialvector B_c mesons can be written as,

$$\Gamma_{\ell\bar{\nu}_\ell} = \frac{G_F^2}{4\pi}|V_{bc}|^2 f_{B_c}^2 M_{B_c}^3 \left(1-\frac{M_\ell^2}{M_{B_c}^2}\right)^2 \left(1+\frac{M_\ell^2}{2M_{B_c}^2}\right) , \quad (12)$$

where $\ell = e, \mu, \tau$, the Fermi constant $G_F = 1.16637 \times 10^{-5}\,\mathrm{GeV}^{-2}$, the CKM matrix element $V_{cb} = 40.6 \times 10^{-3}$, the masses of the leptons $m_e = 0.511 \times 10^{-3}\,\mathrm{GeV}$, $m_\mu = 105.658 \times 10^{-3}\,\mathrm{GeV}$, $m_\tau = 1776.82 \times 10^{-3}\,\mathrm{GeV}$ [4]. We can use the masses and decay constants of the vector and axialvector B_c mesons in Table 1 to obtain the leptonic decay widths,

$$\begin{aligned}
\Gamma_{B_c(1^-)\to e\bar{\nu}_e} &= 2.84^{+0.12+0.76}_{-0.12-0.67} \times 10^{-3}\,\mathrm{eV}, \\
\Gamma_{B_c(1^-)\to \mu\bar{\nu}_\mu} &= 2.84^{+0.12+0.76}_{-0.12-0.68} \times 10^{-3}\,\mathrm{eV}, \\
\Gamma_{B_c(1^-)\to \tau\bar{\nu}_\tau} &= 2.50^{+0.12+0.68}_{-0.11-0.59} \times 10^{-3}\,\mathrm{eV}, \\
\Gamma_{B_c(1^+)\to e\bar{\nu}_e} &= 3.58^{+0.17+1.04}_{-0.15-0.90} \times 10^{-3}\,\mathrm{eV}, \\
\Gamma_{B_c(1^+)\to \mu\bar{\nu}_\mu} &= 3.58^{+0.17+1.04}_{-0.15-0.90} \times 10^{-3}\,\mathrm{eV}, \\
\Gamma_{B_c(1^+)\to \tau\bar{\nu}_\tau} &= 3.21^{+0.16+0.93}_{-0.15-0.81} \times 10^{-3}\,\mathrm{eV}. \quad (13)
\end{aligned}$$

where the uncertainties originate from the uncertainties of the masses and decay constants, respectively. Such tiny leptonic decay widths maybe escape experimental detections.

4 Conclusion

In this article, we study the vector and axialvector B_c mesons with the QCD sum rules, and make predictions for the masses and decay constants, which can be taken as basic input parameters in studying revelent physical physics, such as the semi-leptonic and non-leptonic decays, then calculate the leptonic decay widths, the present predictions can be confronted with the experimental data in the future at the LHC.

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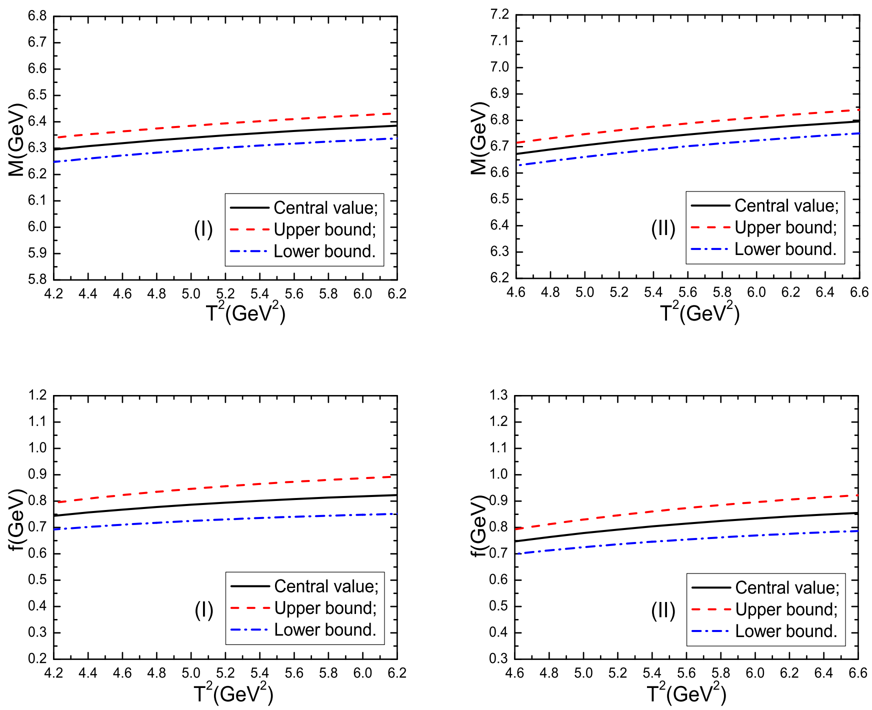

Figure 3: The masses and decay constants of the vector and axialvector B_c mesons with variations of the Borel parameters T^2, where (I) and (II) denote the vector and axialvector B_c mesons, respectively.

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